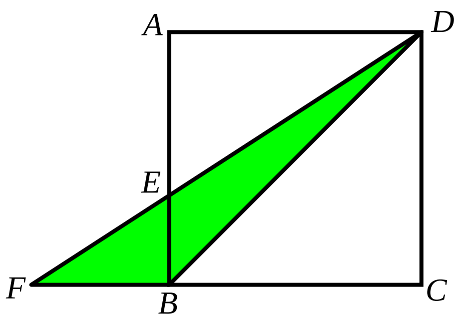
Hong Kong Mathematics Olympiad (2001 – 2002)
Final Event 1 (Individual)
香港数学竞赛 (2001 – 2002)
决赛项目 1 (个人)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 在下图中，$ABCD$ 是一边长为 10cm 的正方形，AEB、FED 及 FBC 为直线，ΔAED 的面积比 ΔFEB 的面积大 10cm^2。若 ΔDFB 的面积为 $P\text{ cm}^2$，求 P 的值。

In the following figure, $ABCD$ is a square of length 10cm. AEB, FED and FBC are straight lines. The area of ΔAED is larger than that of ΔFEB by 10cm^2. If the area of ΔDFB is $P\text{ cm}^2$, find the value of P.

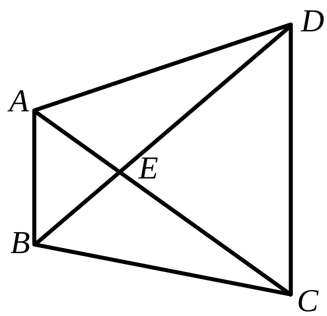


(ii) 一件工程，甲单独需时 90 天完成，而乙则需时 Q 天。若甲、乙二人合做只需 P 天完成，求 Q 的值。

Workman A needs 90 days to finish a task independently while workman B needs Q days for the same task. If they only need P days to finish the task when working together, find the value of Q.

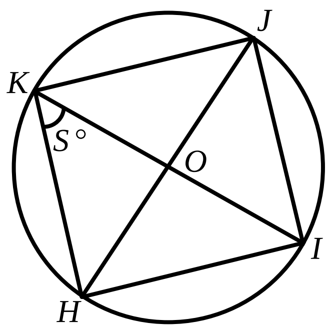
(iii) 在下图中，$AB // CD$，梯形 $ABCD$ 的面积为 $R\text{ cm}^2$。已知 ΔABE 和 ΔCDE 的面积分别为 $Q\text{ cm}^2$ 和 $4Q\text{ cm}^2$，求 R 的值。

In the following figure, $AB // CD$, the area of trapezium $ABCD$ is $R\text{ cm}^2$. Given that the areas of ΔABE and ΔCDE are $Q\text{ cm}^2$ and $4Q\text{ cm}^2$ respectively, find the value of R.



(iv) 在下图中，O 为圆心，HJ 和 IK 为圆的直径以及 $\angle HKI = S°$。已知 $\angle HKI + \angle HOI + \angle HJI = \frac{1}{4}R°$，求 S 的值。

In the following figure , O is the centre of the circle, HJ and IK are diameters and $\angle HKI = S°$. Given that $\angle HKI + \angle HOI + \angle HJI = \frac{1}{4}R°$, find the value of S .



Hong Kong Mathematics Olympiad (2001 – 2002)

Final Event 2 (Individual)

香港数学竞赛 **(2001 – 2002)**

决赛项目 **2 (个人)**

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 已知 $P=\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\cdots+\frac{1}{99\times 100}$，求 P 的值。

Given that $P=\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\cdots+\frac{1}{99\times 100}$, find the value of P.

(ii) 已知 $99Q=P\times\left(1+\frac{99}{100}+\frac{99^2}{100^2}+\frac{99^3}{100^3}+\cdots\right)$，求 Q 的值。

Given that $99Q=P\times\left(1+\frac{99}{100}+\frac{99^2}{100^2}+\frac{99^3}{100^3}+\cdots\right)$, find the value of Q.

(iii) 已知 x 及 R 为实数。若对所有 x，$\frac{2x^2+2Rx+R}{4x^2+6x+3}\le Q$，求 R 的最大值。

Given that x and R are real numbers and $\frac{2x^2+2Rx+R}{4x^2+6x+3}\le Q$ for all x, find the maximum value of R.

(iv) 已知 $S=\log_{144}\sqrt[R]{2}+\log_{144}\sqrt[2R]{R}$，求 S 的值。

Given that $S=\log_{144}\sqrt[R]{2}+\log_{144}\sqrt[2R]{R}$, find the value of S.

Hong Kong Mathematics Olympiad (2001 – 2002)

Final Event 3 (Individual)

香港数学竞赛 (2001 – 2002)

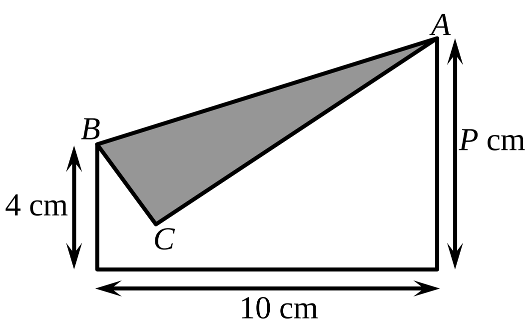
决赛项目 3 (个人)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 将一长方形纸折出以下的图形。若$\triangle ABC$的面积是原长方形纸面积的$\frac{1}{3}$，求P的值。

A rectangular piece of paper is folded into the following figure. If the area of $\triangle ABC$ is $\frac{1}{3}$ of the area of the original rectangular piece of paper, find the value of P.



(ii) 已知$\frac{P}{2}(4^x+4^{-x})-35(2^x+2^{-x})+62=0$。若$Q$是此方程的正整数解，求$Q$的值。

If Q is the positive integral solution of the equation $\frac{P}{2}(4^x+4^{-x})-35(2^x+2^{-x})+62=0$, find the value of Q.

(iii) 设$[a]$表示不大于a的最大整数，例如$[2.5]=2$。若$R=[\sqrt{1}]+[\sqrt{2}]+\cdots+[\sqrt{99Q}]$，求$R$的值。

Let $[a]$ be the largest integer not greater than a. For example, $[2.5]=2$. If $R=[\sqrt{1}]+[\sqrt{2}]+\cdots+[\sqrt{99Q}]$, find the value of R.

(iv) 一个凸多边形，除了内角 A 以外，其它内角的和是 $4R°$。若 $\angle A=S°$，求 S 的值。

In a convex polygon, other than the interior angle A, the sum of all the remaining interior angles is equal to $4R°$. If $\angle A=S°$, find the value of S.

Hong Kong Mathematics Olympiad (2001 – 2002)

Final Event 4 (Individual)

香港数学竞赛 (2001 – 2002)

决赛项目 4 (个人)

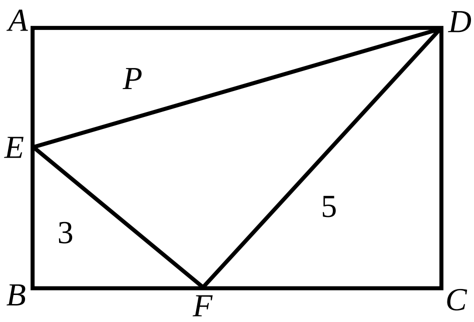
除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 已知 $f(x)=\left(x^2+x-2\right)^{2002}+3$ 及 $f\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)=P$，求 P 的值。

Given that $f(x)=\left(x^2+x-2\right)^{2002}+3$ and $f\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)=P$, find the value of P.

(ii) 在下图中， $ABCD$ 为一长方形。E 和 F 分别是 AB 和 BC 上的点。三角形 AED 、 EBF 和 FCD 的面积分别为 P 、 3 和 5。若 ΔEFD 的面积为 Q，求 Q 的值。

In the following figure, $ABCD$ is a rectangle. E and F are points on AB and BC respectively. The areas of triangles AED, EBF and FCD are P, 3 and 5 respectively. If the area of ΔEFD is Q, find the value of Q.



(iii) 已知 x 和 y 为两正整数。若不等式 $x^2+y^2\leq Q$ 的解 (x, y) 的数目为 R，求 R 的值。

It is given that x and y are positive integers. If the number of solutions (x, y) of the inequality $x^2+y^2\leq Q$ is R, find the value of R.

(iv) 已知 α 和 β 是方程 $x^2 - ax + a - R = 0$ 的两个根，其中 a 为实数。若$(\alpha+1)^2+(\beta+1)^2$ 的最小值为 S，求 S 的值。

It is given that α and β are roots of the equation $x^2 - ax + a - R = 0$, where a is real. If the minimum value of $(\alpha+1)^2+(\beta+1)^2$ is S, find the value of S.